Use the product rule to find the derivative of each of the following functions. Remember your exponent and logarithmic laws, these will help you.

Product Rule	$\frac{d}{dt}(f(t)(g(t)) = f'(t)g(t) + f(t)g'(t)$

In words, the derivative of a product is the product of the derivative of the first function with the second function *plus* the product of the first function with the derivative of the second function.

Example 1:

Differentiate of x^2e^{2x}.

Observe that if $f(x) = x^2$ and $g(x) = e^{2x}$, then,

$$f(x) = x^2 \Longrightarrow f'(x) = 2x$$
$$g(x) = e^{2x} \Longrightarrow g'(x) = 2e^{2x}$$

So then $y' = f'(x)g'(x) + f(x)g'(x) = 2xe^{2x} + x^2 \cdot 2e^{2x} = 2xe^{2x} + 2x^2e^{2x}$.

Example 2:

Differentiate of $t^3 \ln(t+1)$.

Observe that if $f(t) = t^3$ and $g(t) = \ln(t+1)$, then,

$$f(t) = t^3 \Longrightarrow f'(t) = 3t^2$$
$$g(t) = \ln(t+1) \Longrightarrow g'(t) = \frac{1}{t+1}$$

So then $y' = f'(t)g'(t) + f(t)g'(t) = 3t^2\ln(t+1) + t^3 \cdot \frac{1}{t+1} = 3t^2\ln(t+1) + \frac{t^3}{t+1}$.

Example 3:

Differentiate of $\dfrac{e^{2t}}{t^2}$.

Observe that if $f(t) = e^{2t}$ and $g(t) = t^{-2}$, then,

$$f(t) = e^{2t} \Longrightarrow f'(t) = 2e^{2t}$$
$$g(t) = t^{-2} \Longrightarrow g'(t) = -2t^{-3}$$

So then $y' = f'(t)g'(t) + f(t)g'(t) = 2e^{2t} \cdot t^{-2} + e^{2t} \cdot (-2t^{-3}) = 2t^{-2}e^{2t} - 2t^{-3}e^{2t}$.

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1. $f(t) = te^{-2t}$
2. $f(x) = xe^x$
3. $y = t^2(3t+1)^3$
4. $y = 5xe^{x^2}$
5. $y = x\ln(x)$
6. $y = (t^2+3)e^t$
7. $y = (t^3 - 7t^2 + 1)e^t$
8. $z = (3t+1)(5t+2)$
9. $R = 3qe^{-q}$
10. $P = t^2\ln(t)$
11. $f(x) = \frac{x^2+3}{x}$
12. $f(z) = \sqrt{z}e^{-z}$
13. $y = te^{-t^2}$
14. $f(t) = te^{5-2t}$
15. $g(p) = p\ln(2p+1)$
16. $y = x \cdot 2^x$
17. $f(w) = (5w^2+3)e^{w^2}$
18. $w = (t^3+5t)(t^2-7t+2)$
19. $z = (te^{3t} + e^{5t})^9$
20. $f(x) = \frac{x}{e^x}$
21. $z = \frac{1-t}{1+t}$
22. $w = \frac{3z}{1+2z}$
23. $w = \frac{3y+y^2}{5+y}$
24. $y = \frac{1+z}{\ln(z)}$

Answers

1. $f'(t) = e^{-2t} - 2te^{2t}$
2. $f'(x) = e^x + xe^x$
3. $y' = 2t(3t+1)^3 + 9t^2(3t+1)^2$
4. $y' = 5e^{x^2} + 10x^2e^{x^2}$
5. $y' = \ln(x) + 1$
6. $y' = 2te^t + (t^2+3)e^t$
7. $y' = (3t^2 - 14t)e^t + (t^3 - 7t^2 + 1)e^t$
8. $z' = 3(5t+2) + 5(3t+1)$
9. $R' = 3e^{-q} - 3qe^{-q}$
10. $P' = 2t\ln(t) + t$
11. $f'(x) = 2 - (x^2+3)x^{-2}$
12. $f'(x) = \frac{1}{2}z^{-1/2}e^{-z} - \sqrt{z}e^{-z}$
13. $y' = e^{-t^2} - 2t^2e^{-t^2}$
14. $f'(t) = e^{5-2t} - 2te^{5-2t}$
15. $g'(p) = \ln(2p+1) + \frac{p}{2p+1}$
16. $y' = 2^x + \ln(2)x \cdot 2^x$
17. $f'(w) = 10we^{w^2} + 2w(5w^2+3)2^{w^2}$
18. $w' = (3t^2+5)(t^2-7t+2) + (t^3+5t)(2t-7)$
19. $z' = 9(e^{3t}+3te^{3t}+5e^{5t})(te^{3t}+e^{5t})^8$
20. $f'(x) = e^{-x} - xe^{-x}$
21. $z' = -(1+t)^{-1} - (1-t)(1+t)^{-2}$
22. $w' = 3(1+2z)^{-1} - 3z(1+2z)^{-2}$
23. $w' = (3+2y)(5+y)^{-1} - (3y+y^2)(5+y)^{-2}$
24. $y' = (\ln(z))^{-1} - \frac{1+z}{z}(\ln(z))^{-2}$

The End.